Topic 3

Equations and Inequalities

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$

(2)

(b) $(2x - 7)(1 + x) < 0$

(3)

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$

(1)

(Total for Question 1 is 6 marks)

Q2

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total for Question 2 is 6 marks)

Q3

Solve the simultaneous equations

$$y - 3x + 2 = 0$$
$$y^2 - x - 6x^2 = 0$$

(Total for Question 3 is 7 marks)

Q4

A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2,

(b) (i) write down an inequality, in terms of x, for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x.

(1)

(Total for Question 4 is 8 marks)

End of Questions

Pearson

Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks	
(a)	$3x-6<8-2x \rightarrow 5x<14$ (Accept $5x-14<0$ (o.e.))	M1	
	$x<2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone $\leq$)	A1	(2)
(b)	Critical values are $x=\frac{7}{2}$ and -1	B1	
	Choosing "inside" $-1<x<\frac{7}{2}$	M1 A1	(3)
(c)	$-1<x<2.8$	B1ft	(1)
	Accept any exact equivalents to -1, 2.8, 3.5		6

	Notes
(a)	M1 for attempt to rearrange to $kx<m$ (o.e.) Either $k=5$ or $m=14$ should be correct Allow $5x=14$ or even $5x>14$
(b)	B1 for both correct critical values. (May be implied by a correct inequality) M1 ft their values and choose the "inside" region A1 for fully correct inequality (Must be in part (b): do not give marks if only seen in (c)) Condone seeing $x<-1$ in working provided $-1<x$ is in the final answer. e.g. $x>-1,\ x<\frac{7}{2}$ or $x>-1$ "or" $x<\frac{7}{2}$ or $x>-1$ "blank space" $x<\frac{7}{2}$ score M1A0 BUT allow $x>-1$ and $x<\frac{7}{2}$ to score M1A1 (the "and" must be seen) Also $\left(-1,\frac{7}{2}\right)$ will score M1A1 NB $x<-1, x<\frac{7}{2}$ is of course M0A0 and a number line even with "open" ends is M0A0 Allow 3.5 instead of $\frac{7}{2}$
(c)	B1ft for $-1<x<2.8$ (ignoring their previous answers) or ft their answers to part (a) and part (b) provided both answers were regions and not single values. Allow use of "and" between inequalities as in part (b) If their set is empty allow a suitable description in words or the symbol $\varnothing$. Common error: If (a) is correct and in (b) they simply leave their answer as $x<-1$, $x<3.5$ then in (c) $x<-1$ would get B1ft as this is a correct follow through of these 3 inequalities. Penalise use of $\leq$ only on the A1 in part (b). [i.e. condone in part (a)]

Q2.

Question Number	Scheme		Marks
(a)	$6x + x > 1 - 8$	Attempts to expand the bracket and collect x terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $<, \leq, \geq, =$ instead of $>$.	M1
	$x > -1$	Cao	A1
	Do not isw here, mark their final answer.		
			(2)
(b)	$(x+3)(3x-1)[=0]$ $\Rightarrow x = -3$ and $\frac{1}{3}$	M1: Attempt to solve the quadratic to obtain two critical values A1: $x = -3$ **and** $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and 1/3. (Allow 0.333 for 1/3)	M1A1
	$-3 < x < \frac{1}{3}$	M1: Chooses "inside" region (The letter x does not need to be used here) A1ft: Allow $x < \frac{1}{3}$ **and** $x > -3$ or $\left(-3, \frac{1}{3}\right)$ or $x < \frac{1}{3} \cap x > -3$. Follow through their critical values. (must be in terms of x here) Allow all equivalent fractions for -3 and 1/3. Both ($x < \frac{1}{3}$ **or** $x > -3$) and ($x < \frac{1}{3}$, $x > -3$) as a final answer score A0.	M1A1ft
			(4)
			[6]
	Note that use of $\leq$ or $\geq$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs.		

Q3.

Question number	Scheme	Marks
	$y = 3x - 2 \qquad (3x-2)^2 - x - 6x^2 (= 0)$	M1
	$9x^2 - 12x + 4 - x - 6x^2 = 0$ $3x^2 - 13x + 4 = 0$ (or equiv., e.g. $3x^2 = 13x - 4$)	M1 A1cso
	$(3x-1)(x-4) = 0 \qquad x = \ldots \qquad x = \frac{1}{3}$ (or <u>exact</u> equivalent) $x = 4$	M1 A1
	$y = -1 \quad y = 10$ (Solutions need not be "paired")	M1 A1
		[7]
	1st M: Obtaining an equation in x only (or y only). Condone missing "= 0" Condone sign slips, e.g. $(3x+2)^2 - x - 6x^2 = 0$, but <u>not</u> other algebraic mistakes (such as squaring individual terms... see bottom of page). 2nd M: Multiplying out their $(3x-2)^2$, which must lead to a 3 term quadratic, i.e. $ax^2 + bx + c$, where $a \neq 0, b \neq 0, c \neq 0$, <u>and</u> collecting terms. 3rd M: Solving a 3-term quadratic (see general principles at end of scheme). 2nd A: Both values. 4th M: Using an x value, found algebraically, to attempt at least one y value (or using a y value, found algebraically, to attempt at least one x value)... allow b.o.d. for this mark in cases where the value is wrong but working is not shown. 3rd A: Both values. If y solutions are given as x values, or vice-versa, penalise at the end, so that it is possible to score M1 M1A1 M1 A1 M0 A0. <u>"Non-algebraic" solutions:</u> No working, and only one correct solution pair found (e.g. $x = 4, y = 10$): M0 M0 A0 M0 A0 M1 A0 No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1 Both correct solution pairs found, and demonstrated: Full marks <u>Alternative:</u> $x = \frac{y+2}{3} \qquad y^2 - \frac{y+2}{3} - 6\left(\frac{y+2}{3}\right)^2 = 0$ M1 $y^2 - \frac{y+2}{3} - 6\left(\frac{y^2+4y+4}{9}\right) = 0 \qquad y^2 - 9y - 10 = 0$ M1 A1 $(y+1)(y-10) = 0 \qquad y = \ldots \qquad y = -1 \quad y = 10$ M1 A1 $x = \frac{1}{3} \quad x = 4$ M1 A1 <u>Squaring each term in the first equation,</u> e.g. $y^2 - 9x^2 + 4 = 0$, and using this to obtain an equation in x only could score at most 2 marks: M0 M0 A0 M1 A0 M1 A0.	

Pearson

Q4.

Question Number	Scheme	Notes	Marks
	Ignore any references to the units in this question		
(a)	length is '$x+4$'	May be implied	B1
	$x+x+x+4+x+4>19.2 \Rightarrow x>..$	$2x+2(x\pm 4)>19.2$ and proceeds to $x>$ ..... (Accept 'invisible' brackets) Attempts 2 widths + 2 lengths > 19.2 leading to $x>$	M1
	E.g. $x+x+4x+4x>19.2 \Rightarrow x>1.92$ scores B0M1A0		
	$x>2.8$ *	Achieves $x>2.8$ with no errors	A1(*)
			(3)
	Mark parts (b) and (c) together		
(b)(i)	$x(x+4)<21$	Cao	B1
b(ii)	$x^2+4x-21<0$ $(x+7)(x-3)<0 \Rightarrow x=...$	Multiply out lhs, produce 3TQ = 0 and attempt to solve leading to $x=$... according to general guidelines	M1
	Either $-7<x<3$ or $0<x<3$	M1: Attempts the 'inside' for their critical values (may be from a 2TQ here) A1: Accept either $-7<x<3$ or $0<x<3$ or ($x>-7$ **and** $x<3$) or ($x>0$ **and** $x<3$) but not e.g. ($x>-7$, $x<3$) or ($x>-7$ **or** $x<3$) (There is no specific need for them to realise $x>0$)	M1A1
	Note that <u>many</u> candidates stop here		
			(4)
(c)	$2.8<x<3$	Follow through their answers to (a) and (b) Provided "their 3" > 2.8	B1ft
			(1)
			[8]
	Examples		
	$x(x-4)<21 \Rightarrow x^2-4x-21<0$ $(x-7)(x+3)<0,\ x=7, x=-3$ $-3<x<7\ or\ 0<x<7$ $2.8<x<7$ Scores B0M1M1A0B1ft	$x\times 4x<21 \Rightarrow 4x^2-21<0$ $(2x-\sqrt{21})(2x+\sqrt{21})<0,\ x=\pm\frac{\sqrt{21}}{2}$ $-\frac{\sqrt{21}}{2}<x<\frac{\sqrt{21}}{2}\ or\ 0<x<\frac{\sqrt{21}}{2}$ $2.8<x<\frac{\sqrt{21}}{2}$ Scores B0M0M1A0B0	

Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

Q2

Given the simultaneous equations

$$2x + y = 1$$

$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

(Total for Question 2 is 8 marks)

Q3

Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

(Total for Question 3 is 7 marks)

Q4

The equation

$$(k + 3)\,x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total for Question 4 is 7 marks)

Q5

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for Question 5 is 5 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
Q (a)	$5x > 10$, $x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1]	M1, A1 (2)
(b)	$(2x+3)(x-4) = 0$, 'Critical values' are $-\frac{3}{2}$ and 4	M1, A1
	$-\frac{3}{2} < x < 4$	M1 A1ft (4)
(c)	$2 < x < 4$	B1ft (1) [7]
(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0	
(b)	1st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1st A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}$, $x < 4$ and still get this A1 2nd M1 for choosing the "inside region" for their critical values 2nd A1ft follow through their 2 distinct critical values Allow $x > -\frac{3}{2}$ with "or" "," "$\cup$" " " $x < 4$ to score M1A0 but "and" or "$\cap$" score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only	
(c)	B1ft Allow if **a correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) <u>must be regions</u>. Do not follow through single values. If their follow through answer is the empty set accept $\varnothing$ or {} or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see x<4 (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft **Do not award marks for part (b) if only seen in part (c)** Use of $\leq$ instead of < (or $\geq$ instead of >) loses one accuracy mark only, at first occurrence.	

Q2.

Question Number	Scheme	Marks	
(a)	$x^2 - 4k(1-2x) + 5k(=0)$	Makes y the subject from the first equation and substitutes into the second equation $(=0$ not needed here) or eliminates y by a correct method.	M1
	So $x^2 + 8kx + k = 0$ *	Correct completion to printed answer. There must be no incorrect statements.	A1cso
			(2)
(b)	$(8k)^2 - 4k$	M1: Use of $b^2 - 4ac$ (Could be in the quadratic formula or an inequality, $= 0$ not needed yet). There must be some correct substitution but there must be no x's. No formula quoted followed by e.g. $8k^2 - 4k = 0$ is M0. A1: Correct expression. Do not condone missing brackets unless they are implied by later work but can be implied by $(8k)^2 > 4k$ etc.	M1 A1
	$k = \dfrac{1}{16}$ (oe)	Cso (Ignore any reference to $k = 0$) but there must be no contradictory earlier statements. A fully correct solution with no errors.	A1
			(3)
(b) Way 2 Equal roots	$\Rightarrow x^2 + 8kx + k = (x + \sqrt{k})^2$ $\Rightarrow 8k = 2\sqrt{k}$	M1: Correct strategy for equal roots A1: Correct equation	M1A1

Q3.

Question Number	Scheme		Marks
	Either	Or	
	$y^2 = 4 - 4x + x^2$	$x^2 = 4 - 4y + y^2$	M1
	$4(4 - 4x + x^2) - x^2 = 11$ or $4(2-x)^2 - x^2 = 11$	$4y^2 - (4 - 4y + y^2) = 11$ or $4y^2 - (2-y)^2 = 11$	M1
	$3x^2 - 16x + 5 = 0$	$3y^2 + 4y - 15 = 0$ Correct 3 terms	A1
	$(3x-1)(x-5) = 0, \quad x =$	$(3y-5)(y+3) = 0, \quad y = ...$	M1
	$x = \frac{1}{3} \quad x = 5$	$y = \frac{5}{3} \quad y = -3$	A1
	$y = \frac{5}{3} \quad y = -3$	$x = \frac{1}{3} \quad x = 5$	M1 A1
			(7) 7

Notes

1st M: Squaring to give 3 or 4 terms (need a middle term)

2nd M: Substitute to give quadratic in one variable (may have just two terms)

3rd M: Attempt to solve a **3 term** quadratic.

4th M: Attempt to find at least one y value (or x value). (The second variable)

This will be by substitution or by starting again.

If y solutions are given as x values, or vice-versa, penalise accuracy, so that it is possible to score M1 M1A1 M1 A0 M1 A0.

"Non-algebraic" solutions:

No working, and only one correct solution pair found (e.g. $x = 5$, $y = -3$):
M0 M0 A0 M1 A0 M1 A0

No working, and both correct solution pairs found, but not demonstrated:
M0 M0 A0 M1 A1 M1 A1

Both correct solution pairs found, and demonstrated: Full marks are possible (send to review)

Q4.

Question Number	Scheme	Marks
(a)	**Method 1:** Attempts b^2-4ac for $a=(k+3)$, $b=6$ and their c. $c\neq k$ $b^2-4ac=6^2-4(k+3)(k-5)$ $(b^2-4ac=)\quad -4k^2+8k+96$ or $-(b^2-4ac=)\quad 4k^2-8k-96$ (with no prior algebraic errors) As $b^2-4ac>0$, then $-4k^2+8k+96>0$ and so, $k^2-2k-24<0$	M1 A1 B1 A1 *
	Method 2: Considers $b^2>4ac$ for $a=(k+3)$, $b=6$ and their c. $c\neq k$ $6^2>4(k+3)(k-5)$ $4k^2-8k-96<0$ or $-4k^2+8k+96>0$ or $9>(k+3)(k-5)$ (with no prior algebraic errors) and so, $k^2-2k-24<0$ following correct work	M1 A1 B1 A1 * **[4]**
(b)	Attempts to solve $k^2-2k-24=0$ to give $k=$ ($\Rightarrow$ Critical values, $k=6,-4$.) $k^2-2k-24<0$ gives $-4<k<6$	M1 M1 A1 **[3]** **7 marks**
	Notes	
(a)	**Method 1: M1:** Attempts b^2-4ac for $a=(k+3)$, $b=6$ and their c. $c\neq k$ or uses quadratic formula and has this expression under square root. (ignore >0, <0 or $=0$ for first 3 marks) **A1:** Correct expression for b^2-4ac - need not be simplified (may be under root sign) **B1:** Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. If inequality is used early in "proof" may see $4k^2-8k-96<0$ and B1 would be given for $4k^2-8k-96$ correctly stated. **A1:** Applies $b^2-4ac>0$ correctly (**or writes** $b^2-4ac>0$) to achieve the **result given** in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by -1, or taking every term to other side of inequality. **Need conclusion i.e. printed answer.** **Method 2: M1:** Allow $b^2>4ac$, $b^2<4ac$ or $b^2=4ac$ for $a=(k+3)$, $b=6$ and their c. $c\neq k$ **A1:** Correct expressions on either side (ignore $>$, $<$ or $=$). **B1:** Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 **again without error** **A1:** Produces result with no errors seen from initial consideration of $b^2>4ac$.	
(b)	**M1:** Uses factorisation, formula, completion of square method to find two values for k, or finds two **correct** answers with no obvious method **M1:** Their Lower Limit $<k<$ Their Upper Limit Allow the M mark mark for $\leq$. (Allow $k<$ upper and $k>$ lower) **A1:** $-4<k<6$ Lose this mark for $\leq$ Allow (-4, 6) [not square brackets] or $k>-4$ **and** $k<6$ (must be **and** not or) Can also use intersection symbol $\cap$ **NOT** $k>-4$, $k<6$ (M1A0)	
	Special case : **In part (a)** uses $c=k$ instead of $k-5$ - scores 0 . Allow $k+5$ for method marks	
	Special Case: In part (b) Obtaining $-6<k<4$ This is a common wrong answer. Give M1 M1 A0 special case.	
	Special Case: In part (b) Use of x instead of k – M1M1A0	
	Special Case: $-4<k<6$ and $k<-4$, $k>6$ both given is M0A0 for last two marks. Do not treat as isw.	

Q5.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

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Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2 \geqslant 0, (x-4)^2+1 \geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2 > 0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

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$x^2-8x+17$

$=(x-4)^2+1 \geqslant 1$ as $(x-4)^2 \geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2 \geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so $x^2-8x+17>0$

scores M1 A1 A0

correct but not explained

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$

scores M1 A1 A1

correct and explained

Gold Questions

Calculators may not be used

The total mark for this section is 27

Q1

The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

(Total for Question 1 is 7 marks)

Q2

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total for Question 2 is 5 marks)

Q3

The equation $20x^2 = 4kx - 13kx^2 + 2$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$2k^2 + 13k + 20 < 0$$

(4)

(b) Find the set of possible values for k.

(4)

(Total for Question 3 is 8 marks)

Q4

(a) By eliminating y from the equations

$$y = x - 4$$
$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$
$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

(Total for Question 4 is 7 marks)

End of Questions

Pearson

Gold Mark Scheme

Question number	Scheme		Marks	
	(a) $x^2 + kx + (8-k)$ $(=0)$ $8-k$ need not be bracketed		M1	
	$b^2 - 4ac = k^2 - 4(8-k)$		M1	
	$b^2 - 4ac < 0 \Rightarrow k^2 + 4k - 32 < 0$	(*)	A1cso	(3)
	(b) $(k+8)(k-4) = 0$ $k = ...$		M1	
	$k = -8$ $k = 4$		A1	
	Choosing 'inside' region (between the two k values)		M1	
	$-8 < k < 4$ or $4 > k > -8$		A1	(4)
				7

	(a) 1st M: Using the k from the right hand side to form 3-term quadratic in x ('= 0' can be implied), or… attempting to complete the square $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k\ (=0)$ or equiv., using the k from the right hand side. For either approach, condone sign errors. 1st M may be implied when candidate moves straight to the discriminant. 2nd M: Dependent on the 1st M. Forming expressions in k (with no x's) by using b^2 and $4ac$. (Usually seen as the discriminant $b^2 - 4ac$, but separate expressions are fine, and also allow the use of $b^2 + 4ac$. (For 'completing the square' approach, the expression must be clearly separated from the equation in x). If b^2 and $4ac$ are used in the quadratic formula, they must be clearly separated from the formula to score this mark. For any approach, condone sign errors. If the wrong statement $\sqrt{b^2 - 4ac} < 0$ is seen, maximum score is M1 M1 A0. (b) Condone the use of x (instead of k) in part (b). 1st M: Attempt to solve a 3-term quadratic equation in k. It might be different from the given quadratic in part (a). Ignore the use of < in solving the equation. The 1st M1 A1 can be scored if -8 and 4 are achieved, even if stated as $k < -8$, $k < 4$. Allow the first M1 A1 to be scored in part (a). N.B. '$k > -8$, $k < 4$' scores 2nd M1 A0 '$k > -8$ or $k < 4$' scores 2nd M1 A0 '$k > -8$ and $k < 4$' scores 2nd M1 A1 '$k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$' scores 2nd M0 A0 Use of $\leq$ (in the answer) loses the final mark.	

Q2.

Question Number	Scheme	Marks
(a)	[No real roots implies $b^2-4ac<0$.] $b^2-4ac=q^2-4\times2q\times(-1)$	M1
	So $q^2-4\times2q\times(-1)<0$ i.e. $q^2+8q<0$ (*)	A1 cso (2)
(b)	$q(q+8)=0$ or $(q\pm4)^2\pm16=0$	M1
	(q) $=0$ or -8 (2 cvs)	A1
	$-8<q<0$ <u>or</u> $q\in(-8,0)$ <u>or</u> $q<0$ and $q>-8$	A1 ft (3)
		(5 marks)

Q3.

(b)	$2k^2+13k+20=0\Rightarrow k=...$ e.g. $(2k+5)(k+4)=0\Rightarrow k=...$	Attempt to solve the **<u>given</u>** quadratic to find 2 values for k. See general guidance.	M1
	$\Rightarrow k=-\frac{5}{2},-4$	Both correct. May be implied by e.g. $k<-\frac{5}{2}$, $k<-4$ or seen on a sketch. If they use the quadratic formula allow $\frac{-13\pm3}{4}$ for this mark but not $\sqrt{9}$ for 3 and allow e.g. $-\frac{13}{4}\pm\frac{3}{4}$ if they complete the square.	A1
	$-4<k<-\frac{5}{2}$ Allow equivalent values e.g. $-\frac{10}{4}$ i.e. the critical values must be in the form $\frac{a}{b}$ where a and b are integers	M1: Chooses 'inside' region for **their critical values** i.e. Lower Limit $<k<$ Upper Limit or e.g. Lower Limit $\le k\le$ Upper Limit A1: Allow $k\in(-4,-\frac{5}{2})$ or just $(-4,-\frac{5}{2})$ and allow $k>-4$ **and** $k<-2.5$ and $-\frac{5}{2}>k>-4$ but $k>-4$, $k<-\frac{5}{2}$ scores M1A0. $-\frac{5}{2}<k<-4$ is M0A0	M1A1
	Allow working in terms of x in (b) but the answer must be in terms of k for the final mark.		
			(4)
			(8 marks)

Q4.

Question number	Scheme		Marks	
	(a) $2x^2 - x(x-4) = 8$		M1	
	$x^2 + 4x - 8 = 0$	(*)	A1cso	(2)
	(b) $x = \dfrac{-4 \pm \sqrt{4^2 - (4\times 1\times -8)}}{2}$ or $(x+2)^2 \pm 4 - 8 = 0$		M1	
	$x =$ - 2 $\pm$ (any correct expression)		A1	
	$\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ or $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$		B1	
	$y = \left(-2 \pm 2\sqrt{3}\right) - 4$	M: Attempt at least one y value	M1	
	$\underline{x = -2 + 2\sqrt{3},\ \ y = -6 + 2\sqrt{3}}$	$\underline{x = -2 - 2\sqrt{3},\ \ y = -6 - 2\sqrt{3}}$	A1	(5)
				7

(a)	M1	for correct attempt to form an equation in x only. Condone sign errors/slips but attempt at this line must be seen. E.g. $2x^2 - x^2 \pm 4x = 8$ is OK for M1.
	A1cso	for correctly simplifying to printed form. No incorrect working seen. The = 0 <u>is</u> required. **These two marks can be scored in part (b). For multiple attempts pick best.**
(b)	1st M1	for use of correct formula. If formula is not quoted then a fully correct substitution is required. Condone missing $x =$ or just + or − instead of $\pm$ for M1. For completing the square must have as printed or better. If they have $x^2 - 4x - 8 = 0$ then M1 can be given for $(x-2)^2 \pm 4 - 8 = 0$.
	1st A1	for -2 $\pm$ any correct expression. (The $\pm$ is required but $x =$ is not)
	B1	for simplifying the surd e.g. $\sqrt{48} = 4\sqrt{3}$. Must reduce to $b\sqrt{3}$ so $\sqrt{16}\sqrt{3}$ or $\sqrt{4}\sqrt{3}$ are OK.
	2nd M1	for attempting to find at least one y value. Substitution into one of the given equations and an attempt to solve for y.
	2nd A1	for correct y answers. Pairings need <u>not</u> be explicit but they must say which is x and which y. Mis-labelling x and y loses final A1 only.

Platinum Questions

Calculators may not be used

The total mark for this section is 13

1 (*a*) Find the set of values of k for which the equation

$$\frac{x^2+3x+8}{x^2+x-2}=k$$

has no real roots.

(6)

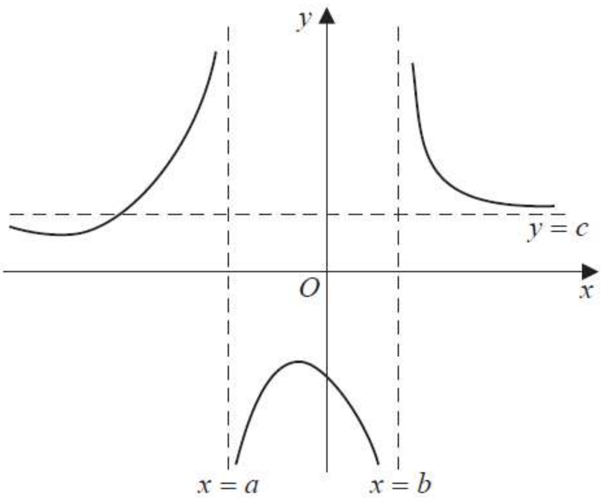

Figure 3

Figure 3 shows a sketch of the curve C_1 with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = \dfrac{x^2+3x+8}{x^2+x-2}=k$

The curve has asymptotes $x = a$, $x = b$ and $y = c$, where a, b and c are integers.

(*b*) Find the value of a, the value of b and the value of c.

(4)

(*c*) Find the coordinates of the points of intersection of C_1 with the line $y = 2$

(3)

(Total for Question 1 is 13 marks)

Platinum Mark Scheme

Qu	Scheme	Mark
7 (a)	$x^2+3x+8=kx^2+kx-2k \quad \Rightarrow \quad 0=(k-1)x^2+(k-3)x-(2k+8)$	M1
	No real roots so "$b^2-4ac<0$" $\Rightarrow (k-3)^2+4(k-1)(2k+8)\ [<0]$	M1
	So $\quad 9k^2+18k-23[<0]$	M1A1
	$(k+1)^2-1-\frac{23}{9}[<0]$	M1
	$k=-1\pm\frac{4\sqrt{2}}{3}$ so $\underline{-1-\frac{4\sqrt{2}}{3}<k<-1+\frac{4\sqrt{2}}{3}}$ (o.e.)	A1cso
		(6)
(b)	$x^2+x-2=(x+2)(x-1)$	M1
	$x=-2,\ x=1 \quad \text{or} \quad a=-2 \ \text{ and } \ b=1$	A1A1
	Division or limits of *x* $\quad$ *y* = 1 $\quad$ or $\quad$ *c* = 1	B1
		(4)
(c)	f(*x*) = 2 $\Rightarrow \quad x^2-x-12=0$	M1
	i.e. $\quad (x-4)(x+3)=0$ so $x=4$ or -3	M1
	Coordinates are **(−3, 2)** and **(4, 2)**	A1
		(3)